This set of slides covers the fundamental concepts of structural dynamics of linear elastic single-degree-of-freedom (SDOF) structures. A separate topic covers the analysis of linear elastic multiple-degree-of-freedom (MDOF) systems. A separate topic also addresses inelastic behavior of structures. Proficiency in earthquake engineering requires a thorough understanding of each of these topics.
This slide lists the scope of the present topic. In a sense, the majority of the material in the topic provides background on the very important subject of response spectra.
Importance in Relation to ASCE 7-05

- Ground motion maps provide ground accelerations in terms of response spectrum coordinates.
- Equivalent lateral force procedure gives base shear in terms of design spectrum and period of vibration.
- Response spectrum is based on 5% critical damping in system.
- Modal superposition analysis uses design response spectrum as basic ground motion input.

The relevance of the current topic to the ASCE 7-05 document is provided here. Detailed referencing to numbered sections in ASCE 7-05 is provided in many of the slides. Note that ASCE 7-05 is directly based on the 2003 NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures, FEMA 450, which is available at no charge from the FEMA Publications Center, 1-800-480-2520 (order by FEMA publication number).
The simple frame is *idealized* as a SDOF mass-spring-dashpot model with a time-varying applied load. The function $u(t)$ defines the displacement response of the system under the loading $F(t)$. The properties of the structure can be completely defined by the mass, damping, and stiffness as shown.

The idealization assumes that all of the mass of the structure can be lumped into a single point and that all of the deformation in the frame occurs in the columns with the beam staying rigid. Represent damping as a simple viscous dashpot common as it allows for a linear dynamic analysis. Other types of damping models (e.g., friction damping) are more realistic but require nonlinear analysis.
Here the equations of motion are shown as a force-balance. At any point in time, the inertial, damping, and elastic resisting forces do not necessarily act in the same direction. However, at each point in time, dynamic equilibrium must be maintained.
This slide (from NONLIN) shows a series of response histories for a SDOF system subjected to a saw-tooth loading. As a result of the loading, the mass will undergo displacement, velocity, and acceleration. Each of these quantities are measured with respect to the fixed base of the structure.

Note that although the loading is discontinuous, the response is relatively smooth. Also, the vertical lines show that velocity is zero when displacement is maximum and acceleration is zero when velocity is maximum.

NONLIN is an educational program for dynamic analysis of simple linear and nonlinear structures. Version 7 is included on the CD containing these instructional materials.
These X-Y curves are taken from the same analysis that produced the response histories of the previous slide. For a linear system, the resisting forces are proportional to the motion. The slope of the inertial-force vs acceleration curve is equal to the mass. Similar relationships exist for damping force vs velocity (slope = damping) and elastic force vs displacement (slope = stiffness).

The importance of understanding and correct use of units cannot be over emphasized.
Here the equations of motion are shown in terms of the displacement, velocity, acceleration, and force relationships presented in the previous slide. Given the forcing function, $F(t)$, the goal is to determine the response history of the system.
Undamped Free Vibration

Equation of motion: \[ m \ddot{u}(t) + k u(t) = 0 \]

Initial conditions: \[ \dot{u}_0 \quad u_0 \]

Assume: \( u(t) = A \sin(\omega t) + B \cos(\omega t) \)

Solution: \[ A = \frac{\dot{u}_0}{\omega} \quad B = u_0 \quad \omega = \sqrt{\frac{k}{m}} \]

\[ u(t) = \frac{\dot{u}_0}{\omega} \sin(\omega t) + u_0 \cos(\omega t) \]

In this unit, we work through a hierarchy of increasingly difficult problems. The simplest problem to solve is undamped free vibration. Usually, this type of response is invoked by imposing a static displacement and then releasing the structure with zero initial velocity. The equation of motion is a second order differential equation with constant coefficients. The displacement term is treated as the primary unknown.

The assumed response is in terms of a sine wave and a cosine wave. It is easy to see that the cosine wave would be generated by imposing an initial displacement on the structure and then releasing. The sine wave would be imposed by initially "shoving" the structure with an initial velocity. The computed solution is a combination of the two effects.

The quantity \( \omega \) is the circular frequency of free vibration of the structure (radians/sec). The higher the stiffness relative to mass, the higher the frequency. The higher the mass with respect to stiffness, the lower the frequency.
This slide shows a computed response history for a system with an initial displacement and velocity. Note that the slope of the initial response curve is equal to the initial velocity \( (v = \frac{du}{dt}) \). If this term is zero, the free vibration response is a simple cosine wave. Note also that the undamped motion shown will continue forever if uninhibited. In real structures, damping will eventually reduce the free vibration response to zero.

The relationship between circular frequency, cyclic frequency, and period of vibration is emphasized. The period of vibration is probably the easiest to visualize and is therefore used in the development of seismic code provisions. The higher the mass relative to stiffness, the longer the period of vibration. The higher the stiffness relative to mass, the lower the period of vibration.
One of the first tasks in any seismic design project is to estimate the period of vibration of the structure. For preliminary design (and often for final design), an empirical period of vibration is used. Section 12.8.2 of ASCE 7-05 provides equations for estimating the period. These equations are listed here.
### Periods of Vibration of Common Structures

<table>
<thead>
<tr>
<th>Structure</th>
<th>Period (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-story moment resisting frame</td>
<td>1.9 sec</td>
</tr>
<tr>
<td>10-story moment resisting frame</td>
<td>1.1 sec</td>
</tr>
<tr>
<td>1-story moment resisting frame</td>
<td>0.15 sec</td>
</tr>
<tr>
<td>20-story braced frame</td>
<td>1.3 sec</td>
</tr>
<tr>
<td>10-story braced frame</td>
<td>0.8 sec</td>
</tr>
<tr>
<td>1-story braced frame</td>
<td>0.1 sec</td>
</tr>
<tr>
<td>Gravity dam</td>
<td>0.2 sec</td>
</tr>
<tr>
<td>Suspension bridge</td>
<td>20 sec</td>
</tr>
</tbody>
</table>

This slide shows typical periods of vibration for several simple structures. Engineers should develop a “feel” for what an appropriate period of vibration is for simple building structures.

For building structures, the formula $T = 0.1 \text{ in}$ is the simplest “reality check.” The period for a 10-story building should be approximately 1 sec. If a computer analysis gives a period of 0.2 sec or 3.0 sec for a 10-story building, something is probably amiss in the analysis.
Damped Free Vibration

Equation of motion:  \[ m \ddot{u}(t) + c \dot{u}(t) + k u(t) = 0 \]

Initial conditions:  \[ u_0 \quad \dot{u}_0 \]

Assume:  \[ u(t) = e^{st} \]

Solution:

\[
 u(t) = e^{-\zeta \omega_D t}
\left[ u_0 \cos(\omega_D t) + \frac{\dot{u}_0 + \zeta \omega u_0}{\omega_D} \sin(\omega_D t) \right]
\]

\[
 \zeta = \frac{c}{2m \omega} = \frac{c}{c_c} \quad \omega_D = \omega \sqrt{1 - \zeta^2}
\]

This slide shows the equation of motion and the response in damped free vibration. Note the similarity with the undamped solution. In particular, note the exponential decay term that serves as a multiplier on the whole response.

Critical damping \((c_c)\) is defined as the amount of damping that will produce no oscillation. See next slide.

The damped circular frequency is computed as shown. Note that in many practical cases \((x < 0.10)\), it will be effectively the same as the undamped frequency. The exception is very highly damped systems.

Note that the damping ratio is often given in terms of % critical.
The concept of critical damping is defined here. A good example of a critically damped response can be found in heavy doors that are fitted with dampers to keep the door from slamming when closing.
Damping in Structures

True damping in structures is NOT viscous. However, for low damping values, viscous damping allows for linear equations and vastly simplifies the solution.

An earlier slide is repeated here to emphasize that damping in real structures is NOT viscous. It is frictional or hysteretic. Viscous damping is used simply because it linearizes the equations of motion. Use of viscous damping is acceptable for the modeling of inherent damping but should be used with extreme caution when representing added damping or energy loss associated with yielding in the primary structural system.
This slide shows some simple damped free vibration responses. When the damping is zero, the vibration goes on forever. When the damping is 20% critical, very few cycles are required for the free vibration to be effectively damped out. For 10% damping, peak is approximately ½ of the amplitude of the previous peak.
Some realistic damping values are listed for structures comprised of different materials. The values for undamaged steel and concrete (upper five lines of table) may be considered as working stress values.
Damping in Structures (3)

Inherent damping

$\xi$ is a structural (material) property independent of mass and stiffness

$\xi_{\text{inherent}} = 0.5$ to $7.0\%$ critical

Added damping

$\xi$ is a structural property dependent on mass and stiffness and damping constant $C$ of device

$\xi_{\text{added}} = 10$ to $30\%$ critical

The distinction between inherent damping and added damping should be clearly understood.
One of the simplest methods to measure damping is a free vibration test. The structure is subjected to an initial displacement and is suddenly released. Damping is determined from the formulas given. The second formula should be used only when the damping is expected to be less than about 10% critical.
The next series of slides covers the response of undamped SDOF systems to simple harmonic loading. Note that the loading frequency is given by the omega term with the overbar. The loading period is designated in a similar fashion.
**Undamped Harmonic Loading (2)**

Equation of motion: \[ m \ddot{u}(t) + k u(t) = p_0 \sin(\omega t) \]

Assume system is initially at rest:

Particular solution: \[ u(t) = C \sin(\tilde{\omega} t) \]

Complimentary solution: \[ u(t) = A \sin(\omega t) + B \cos(\omega t) \]

Solution:

\[
    u(t) = \frac{p_0}{k} \frac{1}{1 - (\tilde{\omega}/\omega)^2} \left( \sin(\tilde{\omega} t) - \frac{\tilde{\omega}}{\omega} \sin(\omega t) \right)
\]

This slide sets up the equation of motion for undamped harmonic loading and gives the solution. We have assumed the system is initially at rest.
Here we break up the response into the steady state response (at the frequency of loading) and the transient response (at the structure’s own natural frequency). Note that the term \( p_0/k \) is the “static” displacement. The dynamic magnifier shows how the dynamic effects may increase (or decrease) the response. This magnifier is a function of the frequency ratio \( \beta \). Note that the magnifier goes to infinity if the frequency ratio \( \beta \) is 1.0. This defines the resonant condition.

In other words, the response is equal to the static response, times a multiplier, times the sum of two sine waves, one in phase with the load and the other in phase with the structure’s undamped natural frequency.
This is a time-history response of a structure with a natural frequency of 4 rad/sec \((f = 2 \text{ Hz}, T = 0.5 \text{ sec})\), and a loading frequency of 2 rad/sec \((f = 1 \text{ Hz}, T = 1 \text{ sec})\), giving a frequency ratio \(\beta\) of 0.5. The harmonic load amplitude is 100 kips. The static displacement is 5.0 inches. Note how the steady state response is at the frequency of loading, is in phase with the loading, and has an amplitude greater than the static displacement. The transient response is at the structure’s own frequency. In real structures, damping would cause this component to disappear after a few cycles of vibration.
In this slide, $\omega$ has been increased to $4\pi$ rad/sec, and the structure is almost at resonance. The steady state response is still in phase with the loading, but note the huge magnification in response. The transient response is practically equal to and opposite the steady state response. The total response increases with time.

If one looks casually at the steady state and transient response curves, it appears that they should cancel out. Note, however, that the two responses are not exactly in phase due to the slight difference in the loading and natural frequencies. This can be seen most clearly at the time 1.75 sec into the response. The steady state response crosses the horizontal axis to the right of the vertical 1.75 sec line while the transient response crosses exactly at 1.75 sec.

In real structures, the observed increased amplitude could occur only to some limit and then yielding would occur. This yielding would introduce hysteretic energy dissipation (apparent damping), causing the transient response to disappear and leading to a constant, damped, steady state response.
This is an enlarged view of the total response curve from the previous slide. Note that the response is bounded within a linear increasing envelope with the increase in displacement per cycle being $2\pi u_s$ times the static displacement.
In this slide, the loading frequency has been slightly increased, but the structure is still nearly at resonance. Note, however, that the steady state response is 180 degrees out of phase with the loading and the transient response is in phase. The resulting total displacement is effectively identical to that shown two slides back.
The loading frequency is now twice the structure’s frequency. The important point here is that the steady state response amplitude is now less than the static displacement.
This plot shows the ratio of the steady state response to the static displacement for the structure loaded at different frequencies. At low loading frequencies, the ratio is 1.0, indicating a nearly static response (as expected). At very high frequency loading, the structure effectively does not have time to respond to the loading so the displacement is small and approaches zero at very high frequency. The resonance phenomena is very clearly shown. The change in sign at resonance is associated with the in-phase/out-of-phase behavior that occurs through resonance.
This is the same as the previous slide but absolute values are plotted. This clearly shows the resonance phenomena.
Damped Harmonic Loading

Equation of motion:

\[ m \ddot{u}(t) + c \dot{u}(t) + k u(t) = p_0 \sin(\omega t) \]

\[ \bar{T} = \frac{2\pi}{\bar{\omega}} = 0.25 \text{ sec} \]

We now introduce damping into the behavior. Note the addition of the appropriate term in the equation of motion.
Damped Harmonic Loading

Equation of motion:

\[ m \ddot{u}(t) + c \dot{u}(t) + k u(t) = p_0 \sin(\omega t) \]

Assume system is initially at rest

Particular solution: \( u(t) = C \sin(\omega t) + D \cos(\omega t) \)

Complimentary solution:

\[ u(t) = e^{-\xi \omega t} \left[ A \sin(\omega_D t) + B \cos(\omega_D t) \right] \]

\[ \xi = \frac{c}{2m\omega} \]

Solution:

\[ u(t) = e^{-\xi \omega t} \left[ A \sin(\omega_D t) + B \cos(\omega_D t) \right] + C \sin(\omega t) + D \cos(\omega t) \]

\[ \omega_D = \omega \sqrt{1 - \xi^2} \]

This slide shows how the solution to the differential equation is obtained. The transient response (as indicated by the \( A \) and \( B \) coefficients) will damp out and is excluded from further discussion.
Damped Harmonic Loading

Transient response at structure’s frequency (eventually damps out)

\[ u(t) = e^{-\xi \omega t} \left[ A \sin(\omega_D t) + B \cos(\omega_D t) \right] + C \sin(\omega t) + D \cos(\omega t) \]

Steady state response, at loading frequency

\[ C = \frac{p_o}{k} \frac{1 - \beta^2}{(1 - \beta^2)^2 + (2\xi \beta)^2} \quad D = \frac{p_o}{k} \frac{-2\xi \beta}{(1 - \beta^2)^2 + (2\xi \beta)^2} \]

This slide shows the \( C \) and \( D \) coefficients of the steady state response. Note that there is a component in phase with the loading (the sine term) and a component out of phase with the loading (the cosine term). The actual phase difference between the loading and the response depends on the damping and frequency ratios.

Note the exponential decay term causes the transient response to damp out in time.
This plot shows the response of a structure at three different loading frequencies. Of significant interest is the resonant response, which is now limited. (The undamped response increases indefinitely.)
For viscously damped structures, the resonance amplitude will always be limited as shown.
A comparison of damped and undamped responses is shown here. The undamped response has a linear increasing envelope; the damped curve will reach a constant steady state response after a few cycles.
This plot shows the dynamic magnification for various damping ratios. For increased damping, the resonant response decreases significantly. Note that for slowly loaded structures, the dynamic amplification is 1.0 (effectively static). For high frequency loading, the magnifier is zero.

Note also that damping is most effective at or near resonance (0.5 < \( \beta < 2.0 \)).
Summary Regarding Viscous Damping in Harmonically Loaded Systems

- For systems loaded at a frequency near their natural frequency, the dynamic response exceeds the static response. This is referred to as *dynamic amplification*.

- An undamped system, loaded at resonance, will have an unbounded increase in displacement over time.

A summary of some of the previous points is provided.
Summary Regarding Viscous Damping in Harmonically Loaded Systems

• Damping is an effective means for *dissipating energy* in the system. Unlike strain energy, which is recoverable, dissipated energy is not recoverable.

• A damped system, loaded at resonance, will have a limited displacement over time with the limit being \((1/2\xi)\) times the static displacement.

• Damping is most effective for systems loaded at or near resonance.

Summary continued.
The discussion will now proceed to general dynamic loading. By general loading, it is meant that no simple mathematical function defines the entire loading history.
General Dynamic Loading
Solution Techniques

- Fourier transform
- Duhamel integration
- Piecewise exact
- Newmark techniques

All techniques are carried out numerically.

There are a variety of ways to solve the general loading problem and all are carried out numerically on the computer. The Fourier transform and Duhamel integral approaches are not particularly efficient (or easy to explain) and, hence, these are not covered here. Any text on structural dynamics will provide the required details.

The piecewise exact method is used primarily in the analysis of linear systems. The Newmark method is useful for both linear and nonlinear systems. Only the basic principles underlying of each of these approaches are presented.
In an earthquake, no actual force is applied to the building. Instead, the ground moves back and forth (and up and down) and this movement induces inertial forces that then deform the structure. It is the displacements in the structure, relative to the moving base, that impose deformations on the structure. Through the elastic properties, these deformations cause elastic forces to develop in the individual members and connections.
Earthquake ground motions usually are imposed through the use of the ground acceleration record or *accelerogram*. Some programs (like Abaqus) may require instead that the ground displacement records be used as input.
In this slide, it is assumed that the ground acceleration record is used as input. The total acceleration at the center of mass is equal to the ground acceleration plus the acceleration of the center of mass relative to the moving base. The inertial force developed at the center of mass is equal to the mass times the total acceleration.

The damping force in the system is a function of the velocity of the top of the structure relative to the moving base. Similarly, the spring force is a function of the displacement at the top of the structure relative to the moving base. The equilibrium equation with the zero on the response history spectrum (RHS) represents the state of the system at any point in time. The zero on the RHS reflects the fact that there is no applied load.

If that part of the total inertial force due to the ground acceleration is moved to the right-hand side (the lower equation), all of the forces on the left-hand side are in terms of the relative acceleration, velocity, and displacement. This equation is essentially the same as that for an applied load (see Slide 8) but the “effective earthquake force” is simply the negative of the mass times the ground acceleration. The equation is then solved for the response history of the relative displacement.
“Simplified” form of Equation of Motion:

\[ m\ddot{u}_r(t) + c\dot{u}_r(t) + ku_r(t) = -m\ddot{u}_g(t) \]

Divide through by \( m \):

\[ \ddot{u}_r(t) + \frac{c}{m}\dot{u}_r(t) + \frac{k}{m}u_r(t) = -\ddot{u}_g(t) \]

Make substitutions:

\[ \frac{c}{m} = 2\xi\omega \quad \frac{k}{m} = \omega^2 \]

Simplified form:

\[ \ddot{u}_r(t) + 2\xi\omega\dot{u}_r(t) + \omega^2 u_r(t) = -\ddot{u}_g(t) \]

In preparation for the development of response spectra, it is convenient to simplify the equation of motion by dividing through by the mass. When the substitutions are made as indicated, it may be seen that the response is uniquely defined by the damping ratio, the undamped circular frequency of vibration, and the ground acceleration record.
For a given ground motion, the response history $u_r(t)$ is function of the structure’s frequency $\omega$ and damping ratio $\xi$. 

\[ \ddot{u}_r(t) + 2\xi\omega \dot{u}_r(t) + \omega^2 u_r(t) = -\ddot{u}_g(t) \]

This restates the point made in the previous slide. A response spectrum is created for a particular ground motion and for a structure with a constant level of damping. The spectrum is obtained by repeatedly solving the equilibrium equations for structures with varying frequencies of vibration and then plotting the peak displacement obtained for that frequency versus the frequency for which the displacement was obtained.
The next several slides treat the development of the 5% damped response spectrum for the 1940 El Centro ground motion record. The “solver” indicated in the slide is a routine, such as the Newmark method, that takes the ground motion record, the damping ratio, and the system frequency as input and reports as output only the maximum absolute value of the relative displacement that occurred over the duration of the ground motion. It is important to note that by taking the absolute value, the sign of the peak response is lost. The time at which the peak response occurred is also lost (simply because it is not recorded).
The Elastic Displacement Response Spectrum

An *elastic displacement response spectrum* is a plot of the peak computed relative displacement, $u_r$, for an elastic structure with a constant damping $\xi$, a varying fundamental frequency $\omega$ (or period $T = 2\pi/\omega$), responding to a given ground motion.

This slide is a restatement of the previous point.
Here, the first point in the response spectrum is computed. For this and all subsequent steps, the ground motion record is the same and the damping ratio is set as 5% critical. Only the frequency of vibration, represented by period $T$, is changed.

When $T = 0.10$ sec (circular frequency = 62.8 radians/sec), the peak computed relative displacement was 0.0543 inches. The response history from which the peak was obtained is shown at the top of the slide. This peak occurred at about 5 sec into the response, but this time is not recorded. Note the high frequency content of the response.

The first point on the displacement response spectrum is simply the displacement (0.0543 inches) plotted against the structural period (0.1 sec) for which the displacement was obtained.
Here the whole procedure is repeated, but the system period is changed to 0.2 sec. The computed displacement history is shown at the top of the slide, which shows that the peak displacement was 0.254 inches. This peak occurred at about 2.5 sec into the response but, as before, this time is not recorded. Note that the response history is somewhat smoother than that in the previous slide.

The second point on the response spectrum is the peak displacement (0.254 inch) plotted against the system period, which was 0.2 sec.
The third point on the response spectrum is the peak displacement (0.622 inch) plotted against the system period, which was 0.3 sec. Again, the response is somewhat “smoother” than before.
The fourth point on the response spectrum is the peak displacement (0.956 inch) plotted against the system period, which was 0.40 sec.
The next point on the response spectrum is the peak displacement (2.02 inches) plotted against the system period, which was 0.50 sec.
The next point on the response spectrum is the peak displacement (3.03 inches) plotted against the system period, which was 0.60 sec. Note that only the absolute value of the displacement is recorded.

The complete spectrum is obtained by repeating the process for all remaining periods in the range of 0.7 through 2.0 sec. For this response spectrum, 2/0.1 or 20 individual points are calculated, requiring 20 full response history analyses. A real response spectrum would likely be run at a period resolution of about 0.01 sec, requiring 200 response history analyses.
This is the full 5% damped elastic displacement response spectrum for the 1940 El Centro ground motion. Note that the spectrum was run for periods up to 4.0 sec. This spectrum was generated using NONLIN.

Note also that the displacement is nearly zero when $T$ is near zero. This is expected because the relative displacement of a very stiff structure (with $T$ near zero) should be very small. The displacement then generally increases with period, although this trend is not consistent. The reductions in displacement at certain periods indicate that the ground motion has little energy at these periods. As shown later, a different earthquake will have an entirely different response spectrum.
If desired, an elastic (relative) velocity response spectrum could be obtained in the same way as the displacement spectrum. The only difference in the procedure would be that the peak velocity computed at each period would be recorded and plotted.

Instead of doing this, the velocity spectrum is obtained in an approximate manner by assuming that the displacement response is harmonic and, hence, that the velocity at each (circular) frequency is equal to the frequency times the displacement. This comes from the rules for differentiating a harmonic function.

Because the velocity spectrum so obtained is not exact, it is called the pseudovelocity response spectrum.

Note that it appears that the pseudovelocity at low (near zero) periods is also near zero (but not exactly zero).
The pseudoacceleration spectrum is obtained from the displacement spectrum by multiplying by the circular frequencies squared. Note that the acceleration at a near zero period is not near zero (as was the case for velocity and displacement). In fact, the pseudoacceleration represents the total acceleration in the system while the pseudovelocity and the displacement are relative quantities.
The pseudoacceleration response spectrum represents the total acceleration of the system, not the relative acceleration. It is nearly identical to the true total acceleration response spectrum for lightly damped structures.

For very rigid systems (with near zero periods of vibration), the relative acceleration will be nearly zero and, hence, the pseudoacceleration, which is the total acceleration, will be equal to the peak ground acceleration.
This slide explains why the pseudoacceleration is equal to the total acceleration. The relative displacement is multiplied by omega to get pseudovelocity. The pseudovelocity then is multiplied by omega to get the total acceleration.
This plot shows total acceleration and pseudoacceleration for a 5% damped system subject to the El Centro ground motion. Note the similarity in the two quantities. The difference in the two quantities is only apparent at low periods.

The difference can be much greater when the damping is set to 10%, 20%, or 30% critical, and the differences can appear in a wider range of periods.
This plot shows relative velocity and pseudovelocity for a 5% damped system subject to the El Centro ground motion. Here, the differences are much more apparent than for pseudoacceleration, and the larger differences occur at the higher periods. The differences will be greater for systems with larger amounts of damping.
The higher the damping, the lower the relative displacement. At a period of 2 sec, for example, going from zero to 5% damping reduces the displacement amplitude by a factor of two. While higher damping produces further decreases in displacement, there is a diminishing return. The % reduction in displacement by going from 5 to 10% damping is much less that that for 0 to 5% damping.
Damping has a similar effect on pseudoacceleration. Note, however, that the pseudoacceleration at a (near) zero period is the same for all damping values. This value is always equal to the peak ground acceleration for the ground motion in question.
Damping Is Effective in Reducing the Response for (Almost) Any Given Period of Vibration

- An earthquake record can be considered to be the combination of a large number of harmonic components.
- Any SDOF structure will be in near resonance with one of these harmonic components.
- Damping is most effective at or near resonance.
- Hence, a response spectrum will show reductions due to damping at all period ranges (except $T = 0$).

Damping is generally effective at all periods (except at $T = 0$). The reason for this is that ground motions consist of a large number of harmonics, each at a different frequency. When a response spectrum analysis is run for a particular period, there will be a near resonant response at that period. Damping is most effective at resonance and, hence, damping will be effective over the full range of periods for which the response spectrum is generated.
Use of an Elastic Response Spectrum

Example Structure

- $K = 500 \text{ k/in}$
- $W = 2,000 \text{ k}$
- $M = \frac{2000}{386.4} = 5.18 \text{ k-sec}^2/\text{in}$
- $\omega = (K/M)^{0.5} = 9.82 \text{ rad/sec}$
- $T = \frac{2\pi}{\omega} = 0.64 \text{ sec}$
- 5% critical damping

At $T = 0.64 \text{ sec}$, displacement = 3.03 in.

This is a simple example of the use of an elastic displacement response spectrum. If the system is assumed to have 5% damping (matching the spectrum) and the system period is known, the peak displacement may be easily computed. Note that the sign of the displacement (positive or negative) and the time that the displacement occurred is not known as this information was discarded when the spectrum was generated.
Use of an Elastic Response Spectrum

**Example Structure**

- $K = 500 \text{ k/in}$
- $W = 2,000 \text{ k}$
- $M = \frac{2000}{386.4} = 5.18 \text{ k-sec}^2/\text{in}$
- $\omega = (K/M)^{0.5} = 9.82 \text{ rad/sec}$
- $T = \frac{2\pi}{\omega} = 0.64 \text{ sec}$
- 5% critical damping

At $T = 0.64 \text{ sec}$, pseudoacceleration = 301 in./sec$^2$

Base shear = $M \times PSA = 5.18(301) = 1559 \text{ kips}$

This is a simple example of the use of an elastic pseudoacceleration response spectrum. If the system is assumed to have 5% damping (matching the spectrum) and the system period and mass are known, the peak base shear may be easily computed. Note that the sign of the shear (positive or negative) and the time that the shear occurred is not known as this information (related to pseudoacceleration) was discarded when the spectrum was generated.
Response spectra often are plotted on four-way log paper. This type of spectrum is often called a “tripartite spectrum” because the displacement, pseudovelocity, and pseudoacceleration are all shown on the same plot. On the plot, pseudovelocity is plotted on the vertical axis. Lines of constant and logarithmically increasing displacement are generated as shown. The use of circular frequency on the horizontal axis is rarely used in practice but is convenient for illustrating the development of the plot.
Lines of constant and logarithmically increasing pseudoacceleration are obtained in a similar manner.
This is a completed spectrum for the 5% damped 1940 El Centro earthquake with maximum acceleration = 0.35g.
Response spectra usually are plotted versus structural period or structural cyclic frequency. This is the same spectrum as shown in the previous slide, but it is plotted versus period.
The use of a single earthquake spectrum in structural design is not recommended for the reasons shown on this slide. The same site experiencing different earthquakes (or different components of the same earthquake) often will have dissimilar spectra.
For a given earthquake, small variations in structural frequency (period) can produce significantly different results.

Note the significant changes (for any given damping value) in the 1.5 sec period range.
The spectra are scaled to 0.4 g with 5% damping. Note the differences.
Because real ground motion spectra are difficult to work with in a design office, a variety of empirical spectra have been generated. One of the earliest of these empirical spectra was developed by Nathan Newmark. The next several slides describe this in detail.

The spectrum used by ASCE 7-05 is simpler than the Newmark spectrum, but explanation of the background of the ASCE 7 spectrum is more difficult. Certain key aspects of the ASCE 7 spectrum are presented in the topic on seismic load analysis.
The Newmark spectrum is based on the following observations:

- The pseudoacceleration at very low periods is exactly equal to the peak ground acceleration.
- The relative displacement at very long periods is exactly equal to the peak ground displacement.
- At intermediate periods, the displacement, pseudovelocity, and pseudoacceleration are equal to the ground values times some empirical constant.
For very low period (high frequency) buildings, the maximum relative displacement will be zero. The maximum acceleration will approach the ground acceleration.
For very high period (low frequency) buildings, the maximum relative displacement will be equal to the maximum ground displacement. The maximum total acceleration will approach zero.
The yellow line shows the maximum recorded ground displacement, velocity, and acceleration from the 1940 El Centro earthquake. These lines clearly form a lower bound to the elastic response spectra. Note how the building response displacements, velocities, and accelerations are amplifications of the ground values. Note also how the amplifications decrease with increased damping.
Newmark’s Spectrum Amplification Factors for Horizontal Elastic Response

<table>
<thead>
<tr>
<th>Damping % Critical</th>
<th>One Sigma (84.1%)</th>
<th>Median (50%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a_a )</td>
<td>( a_v )</td>
</tr>
<tr>
<td>.05</td>
<td>5.10</td>
<td>3.84</td>
</tr>
<tr>
<td>1</td>
<td>4.38</td>
<td>3.38</td>
</tr>
<tr>
<td>2</td>
<td>3.66</td>
<td>2.92</td>
</tr>
<tr>
<td>3</td>
<td>3.24</td>
<td>2.64</td>
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<tr>
<td>5</td>
<td>2.71</td>
<td>2.30</td>
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<tr>
<td>7</td>
<td>2.36</td>
<td>2.08</td>
</tr>
<tr>
<td>10</td>
<td>1.99</td>
<td>1.84</td>
</tr>
<tr>
<td>20</td>
<td>1.26</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Newmark has developed a series of amplification factors to be used in the development of design spectra. These are based on the average of dozens of spectra recorded on firm soil sites for the western United States. Values are shown for the median and median plus one standard deviation.
These are the steps in the development of the Newmark spectrum. Note that actual values are not present.